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# Lifting a Realistic SO(10) Grand Unified Model to Five Dimensions

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## Abstract

It has been shown recently that the problem of rapid proton decay induced by dimension five operators arising from the exchange of colored Higgsinos can be simply avoided in grand unified models where a fifth spatial dimension is compactified on an orbifold. Here we demonstrate that this idea can be used to solve the Higgsino-mediated proton decay problem in any realistic $SO(10)$ model by lifting that model to five dimensions. A particular $SO(10)$ model that has been proposed to explain the pattern of quark and lepton masses and mixings is used as an example. The idea is to break the $SO(10)$ down to the Pati-Salam symmetry by the orbifold boundary conditions. The entire four-dimensional $SO(10)$ model is placed on the physical $SO(10)$ brane except for the gauge fields, the **45** and a single **10** of Higgs fields, which are placed in the five-dimensional bulk. The structure of the Higgs superpotential can be somewhat simplified in doing so, while the Yukawa superpotential and mass matrices derived from it remain essentially unaltered.

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## I. INTRODUCTION

There are several pieces of evidence in favor of supersymmetric grand unification (SUSY GUTs). There is the impressive unification of the gauge couplings of the minimal supersymmetric standard model (MSSM) when extrapolated to high energies. There is the existence of neutrino mass whose magnitude corresponds very well to what is expected from the see-saw mechanism when the right-handed neutrino mass is taken to be of order $M_{GUT}$, the scale at which gauge couplings unify. And there are various features of quark and lepton masses and mixings that can be elegantly accounted for by grand unified symmetries. A well known example of this is the relation $m_b(M_{GUT}) = m_\tau(M_{GUT})$ [1]. Another is that the largeness of the atmospheric neutrino mixing angle may be related to the smallness of the corresponding quark mixing angle $V_{cb}$ by grand unified symmetries in so-called "lopsided" models [2]. It is possible to construct GUT models of quark and lepton mass that are quite simple and predictive.

On the other hand, there are also serious problems with SUSY GUTs, in particular, the problem of doublet-triplet splitting and the closely related problem of dimension five proton decay operators that arise from the exchange of colored Higgsinos. In fact, given better limits on proton decay and better calculations of matrix elements recently, the problem of Higgsino-mediated proton decay has become quite severe [3]. Some of the more successful GUT models of quark and lepton masses (at least in their original forms) can no longer be claimed to be consistent with the improved experimental proton decay limits.

It is noteworthy that the successful features of SUSY GUTs have to do with the gauge interactions and the quark and lepton sectors, whereas all of their difficulties have to do with the Higgs sector. This suggests that SUSY GUTs may require some radically new idea for explaining how gauge symmetries break. It has been known since the rebirth of Kaluza-Klein theories and superstring theory in the 1980's that theories with extra dimensions allow interesting new ways of breaking gauge symmetries and of solving the doublet-triplet splitting problem. Recently, it has been shown that in models with only one or two extra space dimensions orbifold compactification can break grand unified symmetries in such ways as to resolve rather simply both the doublet-triplet splitting and proton decay problems [4,5]. These developments give good reason to suspect that extra dimensions may be the missing ingredient in the idea of grand unification.

The question naturally arises whether these ideas allow one to cure the proton decay problem in actual realistic four-dimensional SUSY GUT models of quark and lepton masses. In this paper we show that they can. We consider a specific published $SO(10)$ model that is quite successful in reproducing the spectrum of quark and lepton masses and mixings — including neutrino mixings — with very few free parameters [6–8]. We show that by embedding this model in five-dimensions not only is the problem of proton decay resolved, but the model is in several ways actually simplified in structure.

The paper is organized as follows. In Sec. II, we briefly review the four-dimensional $SO(10)$ model of quark and lepton masses, emphasizing in particular how the $SO(10)$ symmetry plays a role in explaining many features of the data. We also review the Higgs sector that is needed in realistic $SO(10)$ models, discussing where the general problems lie and what must be done in four dimensions to resolve them. In Sec. III, we show how to embed the model described in Sec. II in five dimensions. We will see that to do this successfully the $SO(10)$ symmetry must be broken not only by orbifold compactification but also by a Higgs in the adjoint representation of $SO(10)$. We will see how this embedding in higher dimension allows the Higgs sector to be significantly simplified.

## II. A REALISTIC FOUR-DIMENSIONAL SO(10) MODEL

### A. The quark and lepton sector

The model we shall study is that proposed in [6–8], which is of the "lopsided" type. The Dirac mass matrices of the quarks and leptons have the form

$$
\begin{aligned}
M_U &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} m_U, \qquad M_D = \begin{pmatrix} \eta & \delta & \delta' \\ \delta & 0 & \sigma+\epsilon/3 \\ \delta' & -\epsilon/3 & 1 \end{pmatrix} m_D, \\
M_\nu^{Dirac} &= \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} m_U, \qquad M_L = \begin{pmatrix} \eta & \delta & \delta' \\ \delta & 0 & -\epsilon \\ \delta' & \sigma+\epsilon & 1 \end{pmatrix} m_D.
\end{aligned}
\tag{1}
$$

The convention being used here is that the left-handed fermion fields multiply the matrices from the right. As explained in Ref. [7], an excellent fit is obtained for the nine quark and charged lepton masses and four CKM mixing parameters with the seven real parameters and complex phase for $\delta'$ after evolution downward from the GUT scale. In addition, the atmospheric neutrino mixing angle is predicted to be very close to maximal, as observed. A simple hierarchical form for the right-handed Majorana mass matrix in terms of three or four additional parameters leads to the large angle solar mixing solution (LMA) that is favored experimentally. This is discussed in Refs. [8,9].

The terms in Eq. (1) come from six operators:

$$
\begin{aligned}
O_{33} &= \mathbf{16}_3\mathbf{16}_3\mathbf{10}_H, \\
O_{23} &= \mathbf{16}_2\mathbf{16}_3\mathbf{10}_H\mathbf{45}_H/M_{GUT}, \\
O'_{23} &= (\mathbf{16}_2\mathbf{16}_H)(\mathbf{16}_3\mathbf{16}'_H)/M_{GUT}, \\
O_{13} &= \mathbf{16}_1\mathbf{16}_3\mathbf{10}''_H, \\
O_{12} &= \mathbf{16}_1\mathbf{16}_2\mathbf{10}'_H, \\
O_{11} &= \mathbf{16}_1\mathbf{16}_1\mathbf{10}_H.
\end{aligned}
\tag{2}
$$

The Higgs fields are denoted by the subscript '$H$'. The quark and lepton multiplets, which are spinors of $SO(10)$, are distinguished by a family label. The adjoint Higgs field $\mathbf{45}_H$ is assumed to have a vacuum expectation value (VEV) proportional to the generator $B-L$. The $\mathbf{10}_H$ has weak-scale VEVs in both the $(1,2,-\frac{1}{2})$ and $(1,2,\frac{1}{2})$ components (under $SU(3)_c\times$

$SU(2)_L \times U(1)_Y$), while the $\mathbf{10}'_H$ and $\mathbf{10}''_H$ have VEVs only in the $(1, 2, -\frac{1}{2})$ component. The $\mathbf{16}_H$ has a GUT-scale VEV in the $(1, 1, 0)$ component, and $\mathbf{16}'_H$ has a weak-scale VEV in the $(1, 2, -\frac{1}{2})$ component.

Many of the features of the mass matrices in Eq. (1) can be understood in group-theoretical terms. The $\epsilon$ entries come from the operator $O_{23}$ in Eq. (2). The factors of $\pm 1$ and $\pm\frac{1}{3}$ that accompany these $\epsilon$ entries arise from the fact that $\langle \mathbf{45}_H \rangle \propto B - L$. Moreover, it can be shown that with $\langle \mathbf{45}_H \rangle \propto B - L$ the $\epsilon$ entries must enter antisymmetrically in flavor, as in fact they do in Eq. (1). (In $O_{23}$ there are two ways to contract the fields to form an $SO(10)$ singlet. One of these contractions is flavor-symmetric, the other flavor-antisymmetric. The flavor-symmetric piece vanishes if $\langle \mathbf{45}_H \rangle \propto B - L$.) The $\sigma$ entries in Eq. (1) come from the operator $O'_{23}$. In the expression defining this operator in Eq. (2) the parentheses mean that the fields inside the parentheses are contracted into a $\mathbf{10}$ of $SO(10)$. From the fact that the VEV of $\mathbf{16}_H$ points in the $SU(5)$-singlet direction, one sees that in $SU(5)$ language $O'_{23} = \mathbf{10}_2 \overline{\mathbf{5}}_3 \overline{\mathbf{5}}_H$. Consequently, the $\sigma$ entries appear in a "lopsided" way and are also transposed between $M_L$ and $M_D$. In Eq. (1), the entries denoted by 1 come from $O_{33}$ and the entries denoted by $\delta$ and $\delta'$ come from $O_{12}$ and $O_{13}$, respectively. All of these are flavor-symmetric since $\mathbf{10}$ is in the symmetric product of $\mathbf{16} \times \mathbf{16}$.

Four of the operators in Eq. (2) are non-renormalizable. These can arise from integration out of real representations of quarks and leptons. For example the operator $\mathbf{16}_2 \mathbf{16}_3 \mathbf{10}_H \mathbf{45}_H / M_{GUT}$ can come from integrating out a vector-like family-antifamily pair, $\mathbf{16} + \overline{\mathbf{16}}$, having the renormalizable couplings $\mathbf{16}_3 \overline{\mathbf{16}}\ \mathbf{45}_H + \mathbf{16}_2 \mathbf{16}\ \mathbf{10}_H + M_{16} \overline{\mathbf{16}}\ \mathbf{16}$. The operator $(\mathbf{16}_2 \mathbf{16}_H)(\mathbf{16}_3 \mathbf{16}'_H)/M_{GUT}$ can come from integrating out a $\mathbf{10}$ of quarks and leptons having the renormalizable couplings $\mathbf{16}_3 \mathbf{10}\ \mathbf{16}'_H + \mathbf{16}_2 \mathbf{10}\ \mathbf{16}_H + M_{10} \mathbf{10}\ \mathbf{10}$. The operator $O_{11}$ involves the integration out of additional vector-like fermions, while the operator $O_{12}$ involves the integration out of other massive Higgs fields, which we do not spell out here. Aside from these last two operators, those listed in Eq. (2) can arise from the following renormalizable terms in the superpotential involving the quark and lepton fields:

$$\begin{aligned} W_{Yukawa} &= \mathbf{16}_3 \mathbf{16}_3 \mathbf{10}_H + \mathbf{16}_3 \overline{\mathbf{16}}\ \mathbf{45}_H + \mathbf{16}_2 \mathbf{16}\ \mathbf{10}_H + M_{16} \overline{\mathbf{16}}\ \mathbf{16} \\ &+ \mathbf{16}_3 \mathbf{10}\ \mathbf{16}'_H + \mathbf{16}_2 \mathbf{10}\ \mathbf{16}_H + M_{10} \mathbf{10}\ \mathbf{10} \\ &+ \mathbf{16}_1 \mathbf{16}_3 \mathbf{10}''_H. \end{aligned} \tag{3}$$

The field content of this model consists of several vectors, spinors, and antispinors of $SO(10)$ together with the adjoint of gauge fields and a single adjoint of Higgs. This adjoint Higgs field plays three very crucial roles. First, it participates in the breaking of $SO(10)$ down to the Standard Model group. Second, it produces the doublet-triplet splitting. To do this it is crucial that its VEV is proportional to $B - L$, since the triplet Higgs that must acquire superheavy masses have $B - L \neq 0$, whereas the doublets that must not get such masses have $B - L = 0$. And, third, the adjoint Higgs, by coupling to the quarks and leptons, introduces the breaking of $SO(10)$ into the quark and lepton masses, so that the "bad" relations $m_s = m_\mu$ and $m_d = m_e$ can be avoided. More precisely, it introduces the factors of $1/3$ into those matrices that are responsible for giving the well-known and successful Georgi-Jarlskog relations $m_\mu(M_{GUT}) \cong 3 m_s(M_{GUT})$ and $m_e(M_{GUT}) \cong m_d(M_{GUT})/3$ [10]. These factors of $1/3$ also require that the VEV of $\mathbf{45}_H$ be proportional to $B - L$.

### B. The Higgs sector

Let us review the Higgs structure that is needed to break $SO(10)$ in four dimensions, both in general and in this model specifically. In general, there are at least four sectors that are needed in the Higgs superpotential: the doublet-triplet-splitting sector, the adjoint sector, the spinor sector, and the adjoint-spinor-coupling sector [11].

The doublet-triplet splitting in $SO(10)$ models in four dimensions must be done by the Dimopoulos-Wilczek mechanism [12,11]. The simplest form of this mechanism assumes the existence of a term $\mathbf{10}_H\mathbf{45}_H\tilde{\mathbf{10}}_H$. (Throughout this paper we will not bother to write the dimensionless coefficients of terms in the superpotential.) There must be two distinct **10**s in this term because the adjoint is in the antisymmetric product of $\mathbf{10}\times\mathbf{10}$. If $\langle\mathbf{45}_H\rangle \propto B-L$, then, for reasons already explained, only the color-triplet fields in the vector Higgs multiplets obtain mass from this term, while the weak doublets all remain massless. Since there are four doublets all told in $\mathbf{10}_H+\tilde{\mathbf{10}}_H$ half of these must be made superheavy to reproduce the MSSM. This can be achieved by the simple term $M(\tilde{\mathbf{10}}_H)^2$. Thus the doublet-triplet-splitting sector has the terms

$$W_{2/3} = \mathbf{10}_H\mathbf{45}_H\tilde{\mathbf{10}}_H + M_1(\tilde{\mathbf{10}}_H)^2. \tag{4}$$

The adjoint sector of the superpotential is responsible for forcing the adjoint Higgs to have a VEV in the $B-L$ direction. The simplest possibility is

$$W_{45} = \mathrm{tr}(\mathbf{45}_H)^4/M_2 - M_3\mathrm{tr}(\mathbf{45}_H)^2. \tag{5}$$

If $\langle\mathbf{45}_H\rangle$ has the form $\mathrm{diag}(a_1,a_2,a_3,a_4,a_5)\otimes i\tau_2$, then the superpotential can be written $W_{45} = \sum_i(a_i^4/M_2 - M_3a_i^2)$. Clearly, the scalar potential is minimized by $a_i = 0$ or $(M_2M_3/2)^{1/2}$. One solution is $\langle\mathbf{45}_H\rangle = (M_2M_3/2)^{1/2}\ \mathrm{diag}(0,0,1,1,1,)\times i\tau_2 = \frac{3}{2}(M_2M_3/2)^{1/2}(B-L)$. The quartic term could be a Planck-scale effect, but then in order to make the vacuum expectation value of the adjoint be of order $M_{GUT}$, the parameter $M_3$ would have to be $O(M_{GUT}^2/M_{Pl})$. On the other hand, if the quartic term arises from integrating out some field whose mass is of order $M_{GUT}$, then that field would have to be a **54** or some larger representation. A singlet would not do, since integrating out a singlet could yield a quartic term in $\mathbf{45}_H$ only of the form $(\mathrm{tr}(\mathbf{45}_H)^2)^2/M_2$. This would leave a continuous degeneracy of all minima satisfying $\sum_i a_i^2 = M_2M_3/2$. Obtaining a satisfactory quartic term for the adjoint is thus a significant difficulty for such $SO(10)$ models in four dimensions. We shall see that a solution to this is quite simple in five-dimensional models.

The spinor sector is required to generate GUT-scale VEVs for the spinor Higgs fields $\mathbf{16}_H+\overline{\mathbf{16}}_H$. These VEVs break $SO(10)$ down to an $SU(5)$ subgroup (this could alternatively be done by $\overline{\mathbf{126}}_H+\mathbf{126}_H$), and the adjoint VEV proportional to $B-L$ further breaks it down to the Standard Model group $SU(3)_c\times SU(2)_L\times U(1)_Y$. It is very easy to construct a superpotential to give the spinors such VEVs. The simplest is

$$W_{16} = (\overline{\mathbf{16}}_H\mathbf{16}_H - M_4^2)\mathbf{1}_{1H}, \tag{6}$$

As explained in Ref. [11] there must be a sector that couples the adjoint Higgs to the spinor Higgs. If there is none, then there is nothing to determine the relative orientation of the adjoint and spinor VEVs. Corresponding to this degeneracy there would be unwanted goldstone bosons. This is a serious difficulty in $SO(10)$ model building in four dimensions since most ways to couple the $\mathbf{45}_H$ to the $\overline{\mathbf{16}}_H + \mathbf{16}_H$ (for example, by the obvious term $\overline{\mathbf{16}}_H \mathbf{45}_H \mathbf{16}_H$) would destabilize the adjoint VEV so that it would no longer exactly satisfy the Dimopoulos-Wilczek condition $\langle \mathbf{45}_H \rangle \propto B - L$. As a result, the doublet Higgs fields in $\mathbf{10}_H$ would acquire GUT-scale mass. One way around this dilemma was suggested in [11] where it was pointed out that if there are three distinct adjoint fields the adjoint-spinor coupling can be done without destabilizing the Dimopoulos-Wilczek form of the adjoint VEV. In Ref. [13] it was then shown that it could be done with only a single adjoint Higgs field, but with additional spinor Higgs fields, if the adjoint-spinor couplings in the superpotential have the form

$$W_{45-16} = \overline{\mathbf{16}}_H(\mathbf{45}_H - \mathbf{1}_{2H})\mathbf{16}'_H + \overline{\mathbf{16}}'_H(\mathbf{45}_H - \mathbf{1}_{3H})\mathbf{16}_H. \tag{7}$$

The primed spinor fields here do not acquire VEVs (at least at the GUT scale, they may at the weak scale). Consequently, the $F$ terms for the adjoint Higgs field and the unprimed spinor Higgs fields do not get contributions from the terms in $W_{45-16}$. The fields $\mathbf{1}_{2H}$ and $\mathbf{1}_{3H}$ are "sliding singlets," whose VEVs are free to adjust to make the $F$ terms of the primed spinor Higgs fields vanish.

The four sectors just discussed are the only ones that are necessary in four-dimensional models based on $SO(10)$ to break $SO(10)$ all the way down to the Standard Model group. However, additional sectors may be necessary for other reasons. For example, in the model of quark and lepton masses we are discussing, as well as in some other published models, it is assumed that there are spinor Higgs that have only $SU(2)_L \times U(1)_Y$-breaking, weak-scale VEVs. The most economical possibility would be for those Higgs to be the primed spinors that appear in $W_{45-16}$. Then to induce the weak-scale VEV we want, one must have terms that mix these primed spinors with the $\mathbf{10}_H$. One possibility is $\overline{\mathbf{16}}_H\overline{\mathbf{16}}_H\mathbf{10}_H$. Then the term $F^*_{\overline{16}_H} F_{\overline{16}_H}$ mixes the $(1, 2, -\frac{1}{2})$ components of $\mathbf{10}_H$ and $\mathbf{16}'_H$. In this model the $\mathbf{10}'_H$ is assumed to have a weak-scale VEV in the $(1, 2, -\frac{1}{2})$ direction, but not the $(1, 2, +\frac{1}{2})$ direction. This can be achieved by introducing terms $\mathbf{10}'_H\mathbf{16}_H\mathbf{16}'_H + M_5\mathbf{10}'_H\mathbf{10}'_H$. Then the VEV of $\mathbf{16}'_H$ in the $(1, 2, -\frac{1}{2})$ direction induces one in $\mathbf{10}'_H$. In this model, then, there is a sector of the superpotential that mixes the vector and spinor Higgs, having the terms

$$W_{10-16} = \overline{\mathbf{16}}_H\overline{\mathbf{16}}_H\mathbf{10}_H + \mathbf{10}'_H\mathbf{16}_H\mathbf{16}'_H + M_5\mathbf{10}'_H\mathbf{10}'_H. \tag{8}$$

A very important consideration in four-dimensional models based on $SO(10)$ is preserving the gauge hierarchy in a natural way. There are two classes of terms that can destroy the gauge hierarchy. Class A consists of operators of the type $M(\mathbf{10}_H)^2$, $M\mathbf{10}_H\tilde{\mathbf{10}}_H$, $(\mathbf{10}_H)^2(\mathbf{45}_H)^2/M_{Pl}$, $\mathbf{10}_H\tilde{\mathbf{10}}_H(\mathbf{45}_H)^2/M_{Pl}$, $(\mathbf{10}_H)^2(\overline{\mathbf{16}}_H\mathbf{16}_H)/M_{Pl}$, etc., which directly produce too large a mass for the MSSM doublets $H_u$ and $H_d$. Such a mass must not be larger than order $M^5_{GUT}/M^4_{Pl}$. Class B consists of operators that destabilize the Dimopoulos-Wilczek form of the VEV of the adjoint Higgs field. If one writes $\langle \mathbf{45}_H \rangle = A(B-L) + BI_{3R}$, then $B/A$ must be less than or about $10^{-13} \sim (M_{GUT}/M_{Pl})^4$. Many terms that directly couple the adjoint Higgs to the spinor Higgs, such as $\overline{\mathbf{16}}_H\mathbf{45}_H\mathbf{16}_H$ and $\overline{\mathbf{16}}_H(\mathbf{45}_H)^2\mathbf{16}_H/M_{Pl}$, fall into this class.

Such operators may be forbidden by a flavor symmetry. For example, in [7] a $U(1) \times Z_2 \times Z_2$ flavor symmetry was shown to protect the gauge hierarchy. The price of such a symmetry is that the field content of the model and the form of the couplings must be somewhat more complicated. For example, the terms in Eqs. (5) and (6) are too simple as they stand. Eq. (5) would imply that $(\mathbf{45}_H)^2$ is neutral under all symmetries, and Eq. (6) would imply the same thing about $\overline{\mathbf{16}}_H \mathbf{16}_H$. Consequently the term $\overline{\mathbf{16}}_H (\mathbf{45}_H)^2 \mathbf{16}_H / M_{Pl}$, which would destroy the gauge hierarchy, would be allowed by all symmetries. In [7] this problem was avoided by replacing the expression in Eq. (6) by one of the form $((\overline{\mathbf{16}}_H \mathbf{16}_H)^2/M^2 - M^2)\mathbf{1}_H$. This allows $\overline{\mathbf{16}}_H \mathbf{16}_H$ and thus $\overline{\mathbf{16}}_H (\mathbf{45}_H)^2 \mathbf{16}_H / M_{Pl}$ to be odd under a $Z_2$. In order to impose the abelian flavor symmetry other complications had to be introduced in [7]. These included (a) replacing most of the dimensionful parameters that appear in Eqs. (3) - (7), and some of the dimensionless couplings, by the VEVs of gauge-singlet Higgs fields, (b) having two **10**s of quarks and leptons instead of just one appearing in the terms in Eq. (3) that give rise to the effective operator $O'_{23}$, (c) having different $\mathbf{10}'_H$ fields appearing in the operators $O_{12}$ and $O_{13}$, and (d) obtaining the operator $O_{11}$ by integrating out an additional spinor-antispinor pair of quarks and leptons.

We will see in the next section that one of the great advantages of embedding the $SO(10)$ model in five-dimensions is that it is no longer necessary to have the Dimopoulos-Wilczek form be highly exact, since the doublet-triplet splitting is produced by the orbifold compactification rather than by the adjoint Higgs field. This means that it is no longer necessary to worry about operators of Class B.

We will not further discuss the details of the model of quark and lepton masses proposed in Refs. [6–8]. Those interested in them can consult those papers. We now turn to the question of whether and how this model can be embedded in five space-time dimensions. The reason for doing so is that as a four-dimensional model it potentially has a serious difficulty with Higgsino-mediated proton decay. To understand why going to five dimensions resolves the proton-decay problem it is useful to review why there is a problem in four dimensions. The Higgs doublets $H_d$ and $H_u$ of the MSSM sit in the grand unified multiplets as follows: $(H_d, H_{\overline{3}}) = \overline{\mathbf{5}}_H \subset \mathbf{10}_H$, and $(H_u, H_3) = \mathbf{5}_H \subset \mathbf{10}_H$. The doublets $H_d$ and $H_u$ do not get mass from the terms in Eq. (4). However, their color-triplet partners, $H_{\overline{3}}$ and $H_3$, do get superheavy masses with the color triplets $\tilde{H}_3$ and $\tilde{H}_{\overline{3}}$ in $\tilde{\mathbf{10}}_H$ from the first term in Eq. (4). In turn, $\tilde{H}_3$ and $\tilde{H}_{\overline{3}}$ get mass with each other from the second term in Eq. (4). Consequently, if the triplets in $\tilde{\mathbf{10}}_H$ are integrated out, one has effectively a mass term linking $H_{\overline{3}}$ to $H_3$. It is this that allows a diagram in which exchange of color-triplet Higgsinos mediates proton decay.

A crucial point in the five-dimensional model that we shall outline in the next section is that the color-triplet Higgsinos $H_{\overline{3}}$ and $H_3$ can get superlarge mass without having a mass term that links them to each other. This happens because they each get a Kaluza-Klein mass that links them not to each other but to Kaluza-Klein modes in the same five-dimensional hyperfield.

## III. LIFTING THE MODEL TO FIVE DIMENSIONS

In the four-dimensional model the Higgs field in the adjoint representation played three crucial roles: it helped break $SO(10)$ down to the Standard Model group, it did the doublet-triplet splitting, and it gave the Clebsch factors of $1/3$ in the quark mass matrices that allowed the model to reproduce the Georgi-Jarlskog relations between quark and lepton masses. In five-dimensional models it has been shown that the first two tasks can be accomplished by orbifold compactification rather than by an adjoint Higgs. It would be exceedingly interesting, therefore, to find a model in which orbifold compactification could also account for the Georgi-Jarlskog factors. In that case, the adjoint Higgs could be dispensed with altogether. However, it is not easy to see how the desired Clebsch factors of $1/3$ can be predicted (rather than merely being accommodated) by orbifold compactification. Therefore, it seems that in extending the model that we have been examining to higher dimension it is necessary to retain an adjoint Higgs field.

In constructing the five-dimensional version of the model we follow the procedure explained in [4,5]. We suppose that the fifth dimension is compactified on a $S^1/(Z_2 \times Z_2')$ orbifold. The $S^1$ is defined by $y \equiv y + 2n\pi R$; the $Z_2$ maps $y \leftrightarrow -y$ and the $Z_2'$ maps $y' \leftrightarrow -y'$, where $y' = y + \pi R/2$. The fundamental region may therefore be taken to be $-\pi R/2 \leq y \leq 0$. Point $O$ at $y = 0$ (the fixed point of $Z_2$) is the "visible brane," while point $O'$ at $y' = 0$ (the fixed point of $Z_2'$) is the "hidden brane". The compactification scale $1/R \equiv M_C$ is assumed to be close to the scale at which the gauge couplings unify, i.e., the GUT scale where $M_{GUT} \sim 10^{16}$ GeV.

The generic bulk field $\phi(x^\mu, y)$, where $\mu = 0, 1, 2, 3$, has definite parity assignment under $Z_2 \times Z_2'$ symmetry. Taking $P = \pm 1$ to be the parity eigenvalue of the field $\phi(x^\mu, y)$ under $Z_2$ transformation and $P' = \pm 1$ to be the parity eigenvalue under the $Z_2'$ transformation, a field with $(P, P') = (\pm, \pm)$ can be denoted $\phi^{PP'}(x^\mu, y) = \phi^{\pm\pm}(x^\mu, y)$. The Fourier series expansion of the fields $\phi^{\pm\pm}(x^\mu, y)$ yields

$$
\begin{aligned}
\phi^{++}(x^\mu, y) &= \tfrac{1}{\sqrt{2^{\delta_{n0}}\pi R}} \textstyle\sum_{n=0}^{\infty} \phi^{++(2n)}(x^\mu) \cos \tfrac{2ny}{R}, \\
\phi^{+-}(x^\mu, y) &= \tfrac{1}{\sqrt{\pi R}} \textstyle\sum_{n=0}^{\infty} \phi^{+-(2n+1)}(x^\mu) \cos \tfrac{(2n+1)y}{R}, \\
\phi^{-+}(x^\mu, y) &= \tfrac{1}{\sqrt{\pi R}} \textstyle\sum_{n=0}^{\infty} \phi^{-+(2n+1)}(x^\mu) \sin \tfrac{(2n+1)y}{R}, \\
\phi^{--}(x^\mu, y) &= \tfrac{1}{\sqrt{\pi R}} \textstyle\sum_{n=0}^{\infty} \phi^{--(2n+2)}(x^\mu) \sin \tfrac{(2n+2)y}{R}.
\end{aligned}
\tag{9}
$$

In the effective theory in four dimensions all of the bulk fields have masses of order $M_C$ except the Kaluza-Klein zero mode $\phi^{++(0)}$ of $\phi^{++}(x^\mu, y)$, which remains massless. Moreover, fields $\phi^{-\pm}(x^\mu, y)$ vanish on the visible brane and fields $\phi^{\pm-}(x^\mu, y)$ vanish on the hidden brane.

In our model, we assume that gauge fields and two multiplets of Higgs fields, $\mathbf{10}_H$ and $\mathbf{45}_H$, exist in the five-dimensional bulk, while the quark and lepton fields and the remaining Higgs fields exist on the visible brane at $O$.

The gauge fields in the bulk are of course in a vector supermultiplet of five-dimensional supersymmetry that is an adjoint representation of $SO(10)$. We will denote it by $\mathbf{45}_g$, where the subscript '$g$' stands for 'gauge'. This vector supermultiplet decomposes into a vector multiplet $V$ and a chiral multiplet $\Sigma$ of $N=1$ supersymmetry in four dimensions. The bulk Higgs fields $\mathbf{10}_H$ and $\mathbf{45}_H$ are in hypermultiplets of five-dimensional supersymmetry. Each hypermultiplet splits into two left-handed chiral multiplets $\Phi$ and $\Phi^c$, having opposite gauge quantum numbers.

Following Dermíšek and Mafi [5], we will assume that the orbifold compactification breaks $SO(10)$ down to the Pati-Salam group $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$, and that the further breaking to the Standard Model group is accomplished by the spinor Higgs fields $\mathbf{16}_H + \overline{\mathbf{16}}_H$ that live on the visible brane through the ordinary four-dimensional Higgs mechanism. Under $G_{PS}$ the $SO(10)$ representations decompose as follows: $\mathbf{45} \to (15,1,1)+(1,3,1)+(1,1,3)+(6,2,2)$; $\mathbf{10} \to (6,1,1)+(1,2,2)$; $\mathbf{16} \to (4,2,1)+(\overline{4},1,2)$; and $\overline{\mathbf{16}} \to (\overline{4},2,1)+(4,1,2)$. With these facts in mind we shall now discuss the transformation of the various fields under the $Z_2 \times Z_2'$ parity transformations.

The first $Z_2$ symmetry (the one we denote as unprimed) is used to break supersymmetry to $N=1$ in four-dimensions. ($N=1$ in five dimensions is equivalent to $N=2$ in four dimensions; so we are breaking half the supersymmetries.) To do this we assume that under $Z_2$ the $V$ is even, $\Sigma$ is odd, $\Phi$ are even, and $\Phi^c$ are odd. The $Z_2'$ is used to break $SO(10)$ down to $G_{PS}$. The $(15,1,1)$, $(1,3,1)$ and $(1,1,3)$ of $\mathbf{45}_g$ and $\mathbf{45}_H$ are taken to be even under $Z_2'$, while the $(6,2,2)$ is taken to be odd. In $\mathbf{10}_H$ the $(1,2,2)$ is taken to be even and the $(6,1,1)$ odd.

All told we have

$$\begin{aligned}
\mathbf{45}_g &= V^{++}_{(15,1,1)} + V^{++}_{(1,3,1)} + V^{++}_{(1,1,3)} + V^{+-}_{(6,2,2)} \\
&+ \Sigma^{-+}_{(15,1,1)} + \Sigma^{-+}_{(1,3,1)} + \Sigma^{-+}_{(1,1,3)} + \Sigma^{--}_{(6,2,2)} \\
\mathbf{45}_H &= \Phi^{++}_{(15,1,1)} + \Phi^{++}_{(1,3,1))} + \Phi^{++}_{(1,1,3)} + \Phi^{+-}_{(6,2,2)} \\
&+ \Phi^{c--}_{(15,1,1)} + \Phi^{c--}_{(1,3,1))} + \Phi^{c--}_{(1,1,3)} + \Phi^{c-+}_{(6,2,2)} \\
\mathbf{10}_H &= \Phi^{++}_{(1,2,2)} + \Phi^{+-}_{(6,1,1)} + \Phi^{c--}_{(1,2,2)} + \Phi^{c-+}_{(6,1,1)}
\end{aligned} \tag{10}$$

Massless zero modes of the Kaluza-Klein towers exist only for fields with $Z_2 \times Z_2'$ parity $++$. The $\Phi^{++}_{(1,2,2)}$ in the $\mathbf{10}_H$ contains the two light doublets of the MSSM. Their color-triplet partners are made superheavy by the orbifold compactification. The $\Phi^{++}_{(15,1,1)}$, $\Phi^{++}_{(1,3,1))}$, and $\Phi^{++}_{(1,1,3)}$ in the $\mathbf{45}_H$ contain zero modes that do not get mass from the compactification. However, these will obtain GUT-scale mass from minimizing the superpotential in the four-dimensional effective theory or will be eaten by gauge bosons and get GUT-scale mass that way.

Having done with the parity assignment for the bulk fields we can turn our attention to the brane physics. On the brane at $O$ we put all of the quark and lepton fields, i.e., not only the three families of spinors $\mathbf{16}_i$ but also the real representations of quarks and leptons that are integrated out to produce the effective operators of the form shown in Eq. (2). This includes the fields shown in Eq. (3). We also place on the brane at $O$ all the Higgs fields, except $\mathbf{10}_H$ and $\mathbf{45}_H$, which live in the bulk.

The $Z_2$ parity of fields that live on the visible brane, (i.e. $\mathbf{16}_i$, $\overline{\mathbf{16}}$, $\mathbf{16}$, $\mathbf{10}$, $\overline{\mathbf{16}}_H$, $\mathbf{16}_H$, $\overline{\mathbf{16}}'_H$, $\mathbf{16}'_H$, and $\mathbf{1}_{aH}$) must be positive. The $Z_2'$ parity assignments must be consistent with those of the components of $\mathbf{45}_g$. For example, the quarks and leptons of the third family must have parities $\mathbf{16}_3 \longrightarrow (4,2,1)_3^{+\pm} + (\overline{4},1,2)_3^{+\mp}$. The superpotential on the visible brane can have couplings between brane fields and bulk fields. For example, we assume there to be a Yukawa term of the form $\mathbf{16}_3\mathbf{16}_3\mathbf{10}_H$. This expression is shorthand for

$$
\begin{aligned}
S_{16_3 16_3 10_H} \;=\; & \int \mathrm{d}^5x \, \tfrac{1}{2}\,[\delta(y) - \delta(y-\pi R)]\,\sqrt{2\pi R}\,\int \mathrm{d}^2\theta \,(4,2,1)_3(\overline{4},1,2)_3\Phi^{++}_{(1,2,2)} \\
+\; & \int \mathrm{d}^5x \, \tfrac{1}{2}\,[\delta(y) - \delta(y-\pi R)]\,\sqrt{2\pi R}\,\int \mathrm{d}^2\theta \,(4,2,1)_3(4,2,1)_3\Phi^{+-}_{(6,1,1)} \qquad (11) \\
+\; & \text{h.c.}
\end{aligned}
$$

Note that the products $(4,2,1)^{+\pm}(\overline{4},1,2)^{+\mp}\Phi^{++}_{(1,2,2)}$ and $(4,2,1)^{+\pm}(4,2,1)^{+\pm}\Phi^{+-}_{(6,1,1)}$ are odd under $Z_2'$ which accounts for the minus sign difference between the two delta functions in both terms, since $Z_2'$ maps $y=0$ onto $y=\pi R$. Even though the $\Phi_{(6,1,1)}$ gets mass of order $M_C$ from the compactification, it couples to the quarks and leptons on the visible brane. None of the symmetries discussed so far would prevent a term of the form $\Phi_{(6,1,1)}\Phi_{(6,1,1)}$ in the superpotential on the visible brane. If such a term were present, it would cause proton decay mediated by the fermionic colored fields in the $\Phi_{(6,1,1)}$. However, such a mass term will be prevented from occurring by the abelian flavor symmetry to be discussed later. (In an ordinary four-dimensional theory, such a mass term or its equivalent is required to make the color triplet Higgs and Higgsinos superheavy; in five-dimensional theories such a mass term is not needed as the triplets get Kaluza-Klein masses from the compactification.)

Using the same shorthand notation, we can state the other couplings of the superpotential on the visible brane. These can be divided into the Yukawa terms of the quarks and leptons and the Higgs self-couplings. The Yukawa terms can be chosen to have the same form as in the four-dimensional version of the model, which was described in the last section. However, the Higgs part of the superpotential can be significantly simplified as a result of the embedding in five dimensions. We saw in the last section that the Higgs superpotential in a four-dimensional $SO(10)$ model has at least four pieces: $W_{Higgs} = W_{2/3} + W_{45} + W_{45-16} + W_{16}$. Let us consider these four pieces in turn.

*The doublet-triplet splitting sector.* In the four-dimensional version of the model, the doublet-triplet-splitting required the existence of two vector Higgs, which were denoted $\mathbf{10}_H$ and $\tilde{\mathbf{10}}_H$, with couplings of the form shown in Eq. (4). However, in the five-dimensional model the doublet-triplet splitting is achieved by the orbifold compactification. Consequently, there is no need for the field $\tilde{\mathbf{10}}_H$ at all, and no need for the terms in Eq. (4). That is, the entire doublet-triplet piece of the superpotential $W_{2/3}$ can be dispensed with.

*The adjoint sector.* In the four-dimensional version of the model it was necessary, in order to obtain the Dimopoulos-Wilczek form of the adjoint VEV, $\langle \mathbf{45}_H \rangle \propto B-L$, to have a non-renormalizable term of the form $\mathrm{tr}(\mathbf{45}_H)^4/M$, as shown in Eq. (5). As noted there, if this term arises as an effective operator from integrating out a superheavy field, that field must be a $\mathbf{54}$ or something larger, whereas if it does not come from integrating out some field, then one would expect the denominator $M$ to be of order $M_{Pl}$. In five-dimensions, the problem of obtaining an adjoint VEV in the $B-L$ direction has a simple solution that involves the physics on the "hidden brane" at $O'$.

The gauge symmetry on the hidden brane is the Pati-Salam group $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$. The components of the adjoint Higgs field $\mathbf{45}_H$ that are non-vanishing on the hidden brane are those with $Z_2' = +$, namely the $\Phi_{(15,1,1)}$, $\Phi_{(1,3,1)}$, and $\Phi_{(1,1,3)}$ of $G_{PS}$, as shown in Eq. (10). This means that there is a superpotential on the hidden brane involving these components:

$$W_{O'} = \mathrm{tr}\Phi^3_{(15,1,1,)} - M\mathrm{tr}\Phi^2_{(15,1,1)} - M'\mathrm{tr}\Phi^2_{(1,3,1)} - M''\mathrm{tr}\Phi^2_{(1,1,3)}. \tag{12}$$

Note that a cubic term is allowed for $\Phi_{(15,1,1)}$, since in $SU(4)_c = SO(6)$ the totally symmetrized product of $15^3$ contains the singlet, whereas that is not true in $SO(10)$. That is why in the four-dimensional $SO(10)$ model with only one adjoint field there had to be a term *quartic* in the adjoint field. In $SO(10)$, only if there are three distinct adjoint fields can one write down a cubic term for them. (The difference between $SO(10)$ and $SO(6)$ in this regard is that in $SO(6)$ there is a rank six antisymmetric tensor that allows one to write $\epsilon_{abcdef}T^{[ab]}T^{[cd]}T^{[ef]}$.) This superpotential gives a scalar potential one of whose (supersymmetric) minima is just $\langle\Phi_{(15,1,1)}\rangle \propto B-L$, since of course $B-L$ is just one of the generators of $SU(4)_c$.

It is interesting that in the five-dimensional version of the model one to some extent explains why the adjoint VEV prefers to point in the $B-L$ direction (as is useful in explaining the pattern of quark and lepton masses, as we saw in the last section). In five-dimensions the splitting of the doublet and triplet Higgs requires that the orbifold compactification break $SO(10)$ down to the Pati-Salam group. And the resulting structure of the superpotential has a term coming from the hidden brane that can drive breaking in the $B-L$ direction.

There is a further technical issue concerning the adjoint sector. It is important that the adjoint Higgs transform non-trivially under some flavor symmetry, otherwise it could be replaced by an explicit $SO(10)$-singlet mass in the terms where it couples to quarks and leptons. Consider, in particular, the second term in Eq. (3), $\mathbf{16}_3\overline{\mathbf{16}}\ \mathbf{45}_H$, which is responsible for the Georgi-Jarlskog factors of $\frac{1}{3}$ and 3 that appear in the mass matrices shown in Eq. (1). If the $\mathbf{45}_H$ has no non-trivial flavor charges, then any flavor symmetry would also allow the term $M\mathbf{16}_3\overline{\mathbf{16}}$. This would replace the factors of 3 by a free parameter, so that the Georgi-Jarlskog factors would no longer be predicted. On the other hand, any non-trivial superpotential for the adjoint Higgs is inconsistent with that field having a $U(1)$ flavor charge. In the four-dimensional models of [6–8] this problem was solved by making $\mathbf{45}_H$ negative under a $Z_2$ flavor symmetry, which is consistent with the form given in Eq. (5). However, such a symmetry would not allow the important cubic term in Eq. (12). An interesting solution to this problem is made possible by the brane structure, namely to introduce a singlet Higgs field $W$, that lives only on the brane at $O'$ and that also has negative parity under the $Z_2$ flavor symmetry. Then instead of the cubic term in Eq. (12) one could have the term $\mathrm{tr}(15,1,1)^3 W/M$. Since the field $W$ lives only on the brane at $O'$, while all the quarks and leptons live on the other brane, there is no way for $W$ to substitute for $\mathbf{45}_H$ in quark and lepton Yukawa terms (which it would be allowed to do by symmetries).

*The adjoint-spinor sector.* In the four-dimensional version of the model it is crucial that the VEV of the adjoint (because of its role in the doublet-triplet splitting) point almost exactly in the $B-L$ direction. Specifically, if $\langle\mathbf{45}_H\rangle = A(B-L) + BI_{3R}$ then $B/A \le 10^{-13}$.

However, since in the five-dimensional version of the model the doublet-triplet splitting is done in another way, which does not involve the adjoint Higgs field, there is no reason why $B/A$ needs to be so small. The only role that $\langle \mathbf{45}_H \rangle \propto B-L$ plays is in explaining features of the quark and lepton masses and mixing, such as the Georgi-Jarlskog factors. Thus, in the five-dimensional version of the model there is no harm in there being deviations at the 10% level of the adjoint VEV from $B-L$. Consequently, the various operators (which we called "Class B" in the last section) that destabilize the Dimopoulos-Wilczek form are not a danger in the five-dimensional model. One thing this means is that the coupling of the adjoint Higgs to the spinor Higgs need not have the very special form shown in Eq. (7), but could in principle have simpler and more obvious forms like $\overline{\mathbf{16}}_H \mathbf{45}_H \mathbf{16}_H$. However, in the specific five-dimensional realization of the model we shall present below, the adjoint-spinor sector has the same form as in Eq. (7).

*The spinor sector.* In the four-dimensional model we saw that the simple form for the spinor sector shown in Eq. (6) leads to problems. In particular, the existence of those terms and of the term $(\mathbf{45}_H)^2$ in Eq. (5) implies that no symmetry prevents terms of the form $\overline{\mathbf{16}}_H \mathbf{16}_H (\mathbf{45}_H)^2$, which destabilize the Dimopoulos-Wilczek form of the adjoint VEV. Thus in [7] a more complicated form for $W_{16}$ was chosen that involved the quartic term $(\overline{\mathbf{16}}_H \mathbf{16}_H)^2$. Here, however, since we do not have to worry about slightly deviating from the Dimopoulos-Wilczek form, the spinor sector can have the simple form in Eq. (6), and indeed will in the five-dimensional model that we will present.

We now present the rest of the details of the five-dimensional model. It very closely parallels the four-dimensional model described in [6–8], but with the simplifications already mentioned. In the four-dimensional model a $U(1) \times Z_2 \times Z_2$ flavor symmetry was employed to control the form of the quark and lepton mass matrices and prevent terms in the Higgs superpotential that would destabilize the gauge hierarchy. Here a slightly simpler $U(1) \times Z_2$ flavor symmetry is enough, essentially because we do not need the doublet-triplet-splitting sector of Eq. (4). Aside from the Higgs field $\tilde{\mathbf{10}}_H$ which is absent here, the fields that are needed are exactly those needed in [6–8]. The Higgs and matter superfields, along with the names previously used in the earlier papers, and their $U(1) \times Z_2$ flavor charges are listed in Tables I and II.

The Higgs superpotential (on the brane at $O$) contains the following terms to be compared only with Eqs. (5)-(8), since there is no analogue of Eq. (4):

$$\begin{aligned} W_{Higgs} \;=\; & -M_3 \mathrm{tr}(\mathbf{45}_H)^2 \\ & + \; (\overline{\mathbf{16}}_H \mathbf{16}_H - M_4^2)\, X \\ & + \; \overline{\mathbf{16}}_H \mathbf{45}_H \mathbf{16}'_H \; P/M + \overline{\mathbf{16}}'_H \mathbf{45}_H \mathbf{16}_H \; P/M \\ & + \; \overline{\mathbf{16}}_H \overline{\mathbf{16}}_H \mathbf{10}_H \; Y'/M + \mathbf{10}_{3H} \mathbf{16}_H \mathbf{16}'_H + \mathbf{10}_{1H} \mathbf{10}_{3H} \; S \\ & + \; \mathbf{10}_{2H} \mathbf{10}_{3H} \; S' + \mathbf{10}_{1H} \mathbf{10}_{1H} \; V_M . \end{aligned} \tag{13}$$

The gauge hierarchy would be endangered by the terms we called class A before, namely those like $M(\mathbf{10}_H)^2$. However, these are eliminated by the $U(1)$ flavor symmetry, just as in [6–8]. The terms that we called class B, which destabilize the minimum $\langle \mathbf{45}_H \rangle \propto B-L$, are not all excluded. However, that is not important for the gauge hierarchy, since the doublet-triplet splitting is not done here by the adjoint Higgs via the Dimopoulos-Wilczek mechanism. Terms that do tend to push $\langle \mathbf{45}_H \rangle$ away from the $B-L$ direction are $\overline{\mathbf{16}}_H \mathbf{45}_H \mathbf{16}'_H \; P/M + \overline{\mathbf{16}}'_H \mathbf{45}_H \mathbf{16}_H \; P/M$. However, if the coefficients of these terms are somewhat small, say a tenth, then the VEV of the adjoint is still close enough to the $B-L$ direction to give satisfactry Georgi-Jarskog factors.

The nature of the massless Higgs doublets at the GUT scale can be determined from the $6 \times 6$ mass matrix for the Higgs multiplets which is easily constructed from the above superpotential. This matrix involves only the brane fields and the zero modes of the bulk fields, not the bulk modes with superlarge K-K masses. One finds the Higgs doublet contributing to the up-type quark and Dirac neutrino mass matrices lies solely in the $\mathbf{10}_H$ representation, i.e., it comes for the K-K zero mode of that field:

$$H_u = 2(\mathbf{10}_H), \tag{14}$$

while the Higgs doublet contributing to the down-type quark and charged lepton mass matrices consists of the linear combination

$$H_d = \bar{2}(\mathbf{10}_H)\cos\gamma + \bar{2}(\mathbf{16}'_H)\sin\gamma\cos\gamma' + \bar{2}(\mathbf{10}_{2H})\sin\gamma\sin\gamma' \tag{15}$$

This interesting feature of $H_d$ allows the possibility of Yukawa coupling unification with a moderate value of $\tan\beta \equiv v_u/v_d \sim 5$, provided $\gamma \sim \pi/2$ and $\gamma' \sim 0$. Note also that the above equations reveal why $\mathbf{10}_{2H}$ can develop a weak-scale VEV only in the $(1, 2, -\frac{1}{2})$ direction as previously assumed.

The light doublets given in Eqs. (14) and (15) have $SU(5)$ partners that are color triplets. These color-triplet partners do not have a superlarge mass term connecting them to each other as they would have to have in a four-dimensional model to make them superheavy; so there is no problem with proton decay mediated by exchange of these colored states. However, these triplets are superheavy because of the K-K masses (there are no K-K zero modes for the triplets in the bulk Higgs field $\mathbf{10}_H$).

The Yukawa superpotential on the brane at $O$ for the charged quarks and leptons has the following form (which should be compared to Eq. (3)):

$$\begin{aligned} W_{Yukawa} \;=\; & \mathbf{16}_3\mathbf{16}_3\mathbf{10}_H + \mathbf{16}_3\overline{\mathbf{16}}\;\mathbf{45}_H + \mathbf{16}_2\mathbf{16}\;\mathbf{10}_H + \mathbf{16}\overline{\mathbf{16}}\;\; P' \\ & + \; \mathbf{16}_3\mathbf{10}_2\mathbf{16}'_H + \mathbf{16}_2\mathbf{10}_1\mathbf{16}_H + \mathbf{10}_1\mathbf{10}_2 Y \\ & + \; \mathbf{16}_1\mathbf{16}_2\mathbf{10}_{1H} + \mathbf{16}_1\mathbf{16}_3\mathbf{10}_{2H} + \mathbf{16}_1\overline{\mathbf{16}}'\; Y' \\ & + \; \mathbf{16}'\overline{\mathbf{16}}'\; S + \mathbf{16}'\mathbf{16}'\mathbf{10}_H + \mathbf{16}_3\mathbf{1}_3\overline{\mathbf{16}}_H + \mathbf{16}_2\mathbf{45}_2\overline{\mathbf{16}}_H \\ & + \; \mathbf{45}_1\mathbf{45}_2 P' + \mathbf{45}_1\mathbf{1}_3\mathbf{45}_H + \mathbf{1}_3\mathbf{1}^c_3 P + \mathbf{1}^c_3\mathbf{1}^c_3 V_M. \end{aligned} \tag{16}$$

where we have omitted additional terms and matter fields which contribute to the Dirac 11 mass matrix elements and the first row and first column of the right-handed Majorana mass matrix. Note that since there is a local $SO(10)$ symmetry on the brane at $O$, the Yukawa superpotential has a full $SO(10)$ invariance. As in [7], the 2-3 sector of the Majorana mass matrix arises from the last six terms in the Yukawas superpotential, where the VEV of $V_M$ violates lepton number by two units. In fact, these terms involving adjoint and singlet representations of the quarks and leptons generate a Majorana mass matrix for the right-handed neutrinos that leads to the LMA solution of the solar neutrino problem via the seesaw mechanism.

## IV. CONCLUSIONS

We have taken a very complete $SO(10)$ model that exists in the literature and shown that it can be transcribed, as it were, to five space-time dimensions in such a way as to preserve its successful features (especially its predictions of quark and lepton masses and mixings) but obviate the problem of proton decay mediated by colored Higgsino exchange, which tends to give too rapid a decay in four-dimensional SUSY GUTs.

The idea is to put all the quarks and leptons and most of the Higgs multiplets on a brane where there is the full local $SO(10)$, so that the structure of the original four-dimensional $SO(10)$ model is left essentially intact, including all the predictions for quark and lepton masses and mixings that rely on the group theoretical constraints imposed on the superpotential by $SO(10)$. But the Higgs doublets of the MSSM come from hypermultiplets in the five-dimensional bulk. This permits the doublet-triplet splitting to be done in such a way as to prevent the dangerous proton decay mediated by the exchange of colored Higgsinos, as has been shown in many papers. To do this, $SO(10)$ is broken by the orbifold compactification down to the Pati-Salam group, with the further breaking down to the Standard Model group done by the conventional four-dimensional Higgs mechanism on the physical $SO(10)$ brane as suggested by Dermíšek and Mafi [5].

Because the doublet-triplet splitting is no longer done by the adjoint Higgs field, as in the original four-dimensional model, it is not important in the five-dimensional model that the "Dimopoulos-Wilczek form" of the adjoint VEV be protected to extremely high accuracy to preserve the gauge hierarchy. This relaxes many of the conditions on the Higgs sector that had to be met in the four-dimensional model to keep the gauge hierarchy natural, and thus allows some simplification of the Higgs sector. Since the adjoint Higgs in the four-dimensional model played a key role in giving a realistic pattern of quark and lepton masses, it must still be present in the five-dimensional version, even though it no longer plays a role in doublet-triplet splitting. By putting the adjoint in the bulk, its VEV can be driven to the desired $B - L$ direction by superpotential terms on the hidden Pati-Salam brane.

The approach described in this paper should be applicable to virtually all realistic four-dimensional $SO(10)$ models. Thus, it provides a way of curing any such model of problems with proton decay coming from dimension five operators without affecting its successful features or its predictions for quark and lepton masses and mixings.

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Table I. Higgs superfields and their $SO(10)$ and $U(1) \times Z_2$ flavor transformation assignments.

| | Previous Label | $SO(10)$ | $U(1)$ | $Z_2$ |
|---|---|---|---|---|
| $\mathbf{45}_H$ | $A$ | $\mathbf{45}$ | $0$ | $-$ |
| $\mathbf{16}_H$ | $C$ | $\mathbf{16}$ | $\frac{3}{2}$ | $-$ |
| $\overline{\mathbf{16}}_H$ | $\overline{C}$ | $\overline{\mathbf{16}}$ | $-\frac{3}{2}$ | $-$ |
| $\mathbf{16}'_H$ | $C'$ | $\mathbf{16}$ | $\frac{3}{2} - p$ | $+$ |
| $\overline{\mathbf{16}}'_H$ | $\overline{C}'$ | $\overline{\mathbf{16}}$ | $-\frac{3}{2} - p$ | $+$ |
| $\mathbf{10}_H$ | $T_1$ | $\mathbf{10}$ | $1$ | $+$ |
| $\mathbf{10}_{1H}$ | $T_0$ | $\mathbf{10}$ | $-2 - p$ | $+$ |
| $\mathbf{10}_{2H}$ | $T'_0$ | $\mathbf{10}$ | $-2$ | $+$ |
| $\mathbf{10}_{3H}$ | $\bar{T}_0$ | $\mathbf{10}$ | $-3 + p$ | $-$ |
| $X$ | $X$ | $\mathbf{1}$ | $0$ | $+$ |
| $P$ | $P$ | $\mathbf{1}$ | $p$ | $+$ |
| $P'$ | | $\mathbf{1}$ | $p$ | $-$ |
| $Y$ | $Y$ | $\mathbf{1}$ | $2$ | $-$ |
| $Y'$ | $Y'$ | $\mathbf{1}$ | $2$ | $+$ |
| $S$ | $S$ | $\mathbf{1}$ | $5$ | $-$ |
| $S'$ | $S'$ | $\mathbf{1}$ | $5 - p$ | $-$ |
| $V_M$ | $V_M$ | $\mathbf{1}$ | $4 + 2p$ | $+$ |

Table II. Matter superfields and their $SO(10)$ and $U(1) \times Z_2$ flavor transformation assignments.

| | $SO(10)$ | $U(1)$ | $Z_2$ |
|---|---|---|---|
| $\mathbf{16}_1$ | $\mathbf{16}$ | $\frac{5}{2}$ | $-$ |
| $\mathbf{16}_2$ | $\mathbf{16}$ | $-\frac{1}{2} + p$ | $-$ |
| $\mathbf{16}_3$ | $\mathbf{16}$ | $-\frac{1}{2}$ | $-$ |
| $\mathbf{16}$ | $\mathbf{16}$ | $-\frac{1}{2} - p$ | $-$ |
| $\overline{\mathbf{16}}$ | $\overline{\mathbf{16}}$ | $\frac{1}{2}$ | $+$ |
| $\mathbf{16}'$ | $\mathbf{16}$ | $-\frac{1}{2}$ | $+$ |
| $\overline{\mathbf{16}}'$ | $\overline{\mathbf{16}}$ | $-\frac{9}{2}$ | $-$ |
| $\mathbf{10}_1$ | $\mathbf{10}$ | $-1 - p$ | $+$ |
| $\mathbf{10}_2$ | $\mathbf{10}$ | $-1 + p$ | $-$ |
| $\mathbf{1}_3$ | $\mathbf{1}$ | $2$ | $+$ |
| $\mathbf{1}^c_3$ | $\mathbf{1}$ | $-2 - p$ | $+$ |
| $\mathbf{45}_1$ | $\mathbf{45}$ | $-2$ | $-$ |
| $\mathbf{45}_2$ | $\mathbf{45}$ | $2 - p$ | $+$ |